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Tachyon condensation and universality of DBI action

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Abstract: We show that a low-energy action for massless fluctuations around a tachyonic soliton background representing a codimension one D-brane coincides with the Dirac-Born-Infeld action. The scalar modes which describe transverse oscillations of the D-brane are translational collective coordinates of the soliton. The appearance of the DBI action is a universal feature independent of details of a tachyon effective action, provided it has the structure implied by the open string sigma model partition function.

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Contents

1. Introduction

In the original perturbative string-theory description D-branes [1] are specified by boundary conditions on the open strings ending on them. Their collective coordinates are identified with massless modes of the open strings, and the "acceleration-independent" part of their action — the DBI action [2] — can be derived (using T-duality) as a reduction of Born-Infeld action (directly from the open string sigma model partition function [3] or from conformal invariance condition [4], see [2, 5, 6]).

Supersymmetric D-branes appear also in another guise as black-hole type solitons of $D = 10$ type-II supergravity [7, 8] which carry Ramond-Ramond charges [9]. Their collective coordinates, determined by the massless fluctuation modes in these backgrounds, can be related to the parameters of spontaneously broken gauge symmetries [10]. With an appropriate non-linear parametrization of the supergravity fields in terms of the collective coordinates, one should be able to derive the corresponding DBI actions directly from the type-II supergravity action.[1]

Recently, a new, third, description of D-branes as solitons was suggested [11, 12]. Arguments supporting the proposal that D-branes can be interpreted as tachyonic solitons were given using a variety of approaches: boundary conformal field theory (see, e.g., [13]), Witten's open string field theory [14] (see, e.g., [15]), noncommutative field theory obtained in large $B_{\mu\nu}$ limit (see, e.g., [16]) and simple low-energy effective lagrangian models with specific tachyon potentials [17, 18, 19].

[1] To a large extent this should follow essentially from reparametrization invariance, implying that the action of a boosted 11-d Schwarzschild black hole should be $M \int dt \sqrt{1 - v^2}$, and T- and S- dualities of type-II supergravities.

Remarkably, the latter models were shown to follow from the boundary string field theory (BSFT) [20, 21] in [22]–[25]. The BSFT approach, consistently restricted to the lowest-level (renormalizable) tachyon and vector couplings, is essentially the same as the off-shell sigma model approach [26, 3, 27] being based on the disc partition function of the open string sigma model. The partition function encodes the information not only about the beta-functions but also about the field space metric which relates them to field equations of motion, and thus allows to smoothly interpolate between the standard perturbative tachyon vacuum and a new non-trivial vacuum at the minimum of the tachyon potential.[2] One finds that there are solitonic solutions corresponding to D-branes of lower dimensions [17, 18], and that the descent relations between D-brane tensions hold exactly [23, 25]. These results have been generalized to include a background gauge field [27],[29]–[33].[3]

From the boundary sigma model point of view, one starts with a conformal theory with d Neumann boundary conditions in the UV and adds relevant (tachyon) perturbations driving the theory to an IR fixed point that corresponds to a new (stable or unstable) vacuum with $(d-1-p)$ Dirichlet boundary conditions. The IR fixed point is then interpreted as a closed string vacuum with a Dp-brane. Given that the tension of a Dp-brane is correctly reproduced [23, 25], a further crucial test is to find the spectrum and the effective action for light modes on a Dp-brane obtained as a result of tachyon condensation.

The aim of the present paper is to show that (the "acceleration-independent" part of) the action for the massless scalar modes Φ on the tachyonic soliton and the massless U(1) vector field A is indeed the standard DBI action.

In the context of the Sen's proposal to describe D-branes as open string field theory solitons the massless modes representing transverse D-brane fluctuations appear as collective coordinates. The presence of these massless scalars is a general phenomenon independent of details of an effective field theory action, and is a consequence of spontaneous breaking of the translational symmetry [34]. The existence of a massless vector mode on the soliton is related to the fact that the tachyon is coupled to the (abelian) open string vector field in a non-minimal way, i.e. only through the field strength $F_{\mu\nu}$.[4] The general structure of the dependence of the effective action on $F_{\mu\nu}$ and its coupling to the tachyon T is dictated by the open

[2] A simple consequence of the sigma model approach to the tachyon condensation is the background independence of the tachyon potentials discussed in the framework of Witten's open string field theory in [28]: the zero mode of the tachyon field which is the only argument of the potential does not feel any closed string background.

[3] Since a constant field strength $F_{\mu\nu}$ enters the actions in the same way as a constant $B_{\mu\nu}$-field, the sigma model approach should provide a natural explanation for some of the results for noncommutative tachyon condensation in [16].

[4] Note that one does not find massless vector modes and DBI actions for the collective coordinates for brane solitons in familiar scalar-vector field-theory systems where scalars are coupled to vector fields in a minimal way.

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string sigma model [3, 4, 27]: the action is an integral of a product of the Born-Infeld lagrangian for the vector field and some function of T and its derivatives contracted with functions of $F_{\mu\nu}$. This structure is the only essential assumption one needs to show that the scalar and vector zero modes always combine into the standard DBI action, irrespective of all other details of the effective action.[5]

While the universality of the DBI action holds irrespective of the presence of a tachyonic mode in the spectrum of fluctuations around the soliton, the form of an action for the tachyonic mode[6] does depend on details of an effective string field theory action (the action for the tachyonic mode on D24-brane was shown [35] to have the same form as the one on the D25-brane).

We should add that while finding the DBI action for the D-branes described by tachyonic solitons was expected — after all, the required information is contained in the open string sigma model which gives both, the effective action for the original brane and the action for massless fluctuations of the solitonic brane — to see how this derivation works in detail seems quite instructive. In particular, this clarifies how the brane collective coordinates should appear in the string world sheet action which may be relevant for off-shell aspects of D-branes (cf. [36]).

In section 2 we shall present a derivation of the DBI action from an effective field theory action with the structure implied by the open string sigma model. In section 3 we shall discuss how one can see the same directly in the world-sheet approach, identifying the boundary couplings of the D-brane collective coordinates.

2. Effective field theory picture

2.1 Tachyon-vector actions

The string sigma model approach to field theory describing dynamics of an unstable D25-brane in bosonic string theory or a non-BPS D9-brane in type-IIA string theory leads to an effective action for the tachyon and vector gauge fields of the following general form (for a review, see [27])[7]

$$S = \int d^d x \sqrt{\det(\delta_{\mu\nu} + F_{\mu\nu})}\ \mathcal{L}(T, F, \partial T, \partial F, \partial\partial T, \ldots)\,. \tag{2.1}$$

[5] The precise value of the tension of the resulting Dp-brane does depend of course on details of an effective string field theory action. To reproduce the expected tension one should compute the disc partition at the proper conformal point, or minimize the "trial actions" depending on a finite number of parameters obtained in the BSFT framework in [23, 25].

[6] The tachyonic mode is always present in open bosonic string theory context, and is absent on the kink solution describing condensation of an unstable D9-brane tachyon to produce a D8-brane, which is a stable BPS-object in type-IIA theory.

[7] In the units we shall use $\alpha' = 2$, and our normalizations for T and A_μ are given in section 3.

The tachyon field couples to the vector field through its strength, i.e. through the combinations (and their derivatives)[8] the

$$G^{\mu\nu} = \left(\frac{1}{1+F}\right)^{(\mu\nu)} = \left(\frac{1}{1-F^2}\right)^{\mu\nu}, \qquad H^{\mu\nu} = \left(\frac{1}{1+F}\right)^{[\mu\nu]} = -\left(\frac{F}{1-F^2}\right)^{\mu\nu}, \tag{2.2}$$

where $G^{\mu\nu}$ plays the role of an effective metric as seen by open string excitations in the presence of a constant gauge field background [4].

We will be interested in deriving an effective action for massless modes on a soliton solution up to "acceleration-dependent" (i.e. second and higher derivative) terms. For that reason we may restrict our consideration to the terms in S which do not depend on derivatives of $F_{\mu\nu}$. We will study only codimension one D-branes, i.e. soliton solutions represented by the tachyon field depending only on one coordinate x_1 and the vector field having constant field strength $F_{\mu\nu}$. As will be shown below, such backgrounds satisfy the equations of motion for the vector field only if $F_{1i} = 0$ ($i = 2, \ldots, 10$), i.e., if the matrices in (2.2) obey $G^{1i} = H^{1i} = 0$.

Since in deriving the effective action for massless modes we drop their second and higher derivatives, none of the possible $H^{\mu\nu}$ dependent terms in (2.1) contribute to the effective action, i.e. we are allowed to use an action which depends only on $G^{\mu\nu}$ and T and its derivatives of any order, i.e.

$$S = \int d^d x \sqrt{\det(\delta_{\mu\nu} + F_{\mu\nu})}\ \mathcal{L}(G^{\mu\nu}(F), T, \partial T, \partial\partial T, \ldots)\,. \tag{2.3}$$

Though any action of the form (2.3) will lead to the DBI action for the massless soliton modes, it is useful to recall several explicit actions of that type which were previously discussed in the context of tachyon condensation. The low-energy two-derivative effective action for a D25-brane in open bosonic string theory is (2.3) with $\mathcal{L}$ given by [33, 27]

$$\mathcal{L}_{25} = T_{25}\, \mathrm{e}^{-T}\ (1+T)(1 + G^{\mu\nu}\partial_\mu T \partial_\nu T)\,. \tag{2.4}$$

The corresponding expression for the non-BPS D9-brane in type-IIA theory is [25, 27]

$$\mathcal{L}_9 = T_9\ \mathrm{e}^{-\frac{1}{4}T^2}\left(1 + G^{\mu\nu}\partial_\mu T\partial_\nu T + \frac{1}{2}\log\left(\frac{4}{\mathrm{e}}\right) G^{\mu\nu}T^2\partial_\mu T\partial_\nu T\right). \tag{2.5}$$

The actions (2.4) and (2.5) are obtained in special schemes chosen to reproduce the standard values of the tachyon masses around the perturbative $T = 0$ vacua. The actions (2.4) and (2.5) (or similar model actions in [17, 18]) have soliton solutions representing codimension one D-branes, but they do not reproduce the expected D-brane tensions. However, one can find "interpolating" actions which coincide

[8]The case when the starting point is a Dp-brane with a non-zero number of transverse dimensions represented by massless scalars is obtained by "dimensionally reducing" $F_{\mu\nu}$ matrix.

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with (2.4) and (2.5) in the two-derivative approximation and lead to the correct D-brane tensions. In particular, for a non-BPS D9-brane such action has [25, 32, 27]

$$\mathcal{L}_9 = T_9\, \mathrm{e}^{-\frac{1}{4}T^2} \left(\sqrt{\pi} \frac{\Gamma(1 + G^{\mu\nu}\partial_\mu T \partial_\nu T)}{\Gamma(1/2 + G^{\mu\nu}\partial_\mu T \partial_\nu T)} + \log\left(\frac{4}{\mathrm{e}}\right) G^{\mu\nu} T \partial_\mu \partial_\nu T \right). \qquad (2.6)$$

Here the first term can be shown to coincide with the action found in [25, 32], and the scheme-dependent coefficient of the last term is chosen to reproduce the correct value of the tachyon mass at $T = 0$ [27].

The action (2.3), (2.6) is given by the open superstring partition function on a disc. In the sigma model approach the tachyon coupling at the boundary of the disc can be represented as

$$T(x(\tau)) = T(x) + \xi^\mu(\tau)\partial_\mu T(x) + \frac{1}{2}\xi^\mu(\tau)\xi^\nu(\tau)\partial_\mu\partial_\nu T(x) + \cdots, \qquad (2.7)$$

where τ is the angular coordinate on the boundary of the disc, and $x(\tau) = x + \xi(\tau)$, $\int d\tau \xi(\tau) = 0$. The derivatives of the tachyon field are then treated as independent, and the partition function is computed exactly in $\partial_\mu T$, and up to the first order in $\partial_\mu\partial_\nu T$.

2.2 Collective coordinates and DBI action

We shall assume that the equations for T and A_μ following from (2.3) have a soliton solution with

$$T = T(x_1)\,, \qquad F_{\mu\nu} = \mathrm{const}\,. \qquad (2.8)$$

Following Sen [11], we shall identify the solution with a D$(d-2)$-brane. For example, for model two-derivative actions one finds the kink $T(x) = ux_1$ [18, 25] in the case of the type-IIA D9-brane, and the lump $T(x) = ux_1^2$ [17, 23] in the case of the bosonic D25-brane. These solutions represent exact conformal field theories in the limit $u \to \infty$ [13, 23, 25].

It is straightforward to see that for any solution for T found for $F_{\mu\nu} = 0$ there is a corresponding solution for $F_{\mu\nu} = \mathrm{const}$. Since the vector potential A_μ is a dynamical field, one is still to check that its equation is satisfied for the above background. To do that we shall restrict ourselves to the simplest case when the action can be put in the form

$$S = \int d^d x \sqrt{\det(\delta_{\mu\nu} + F_{\mu\nu})}\, \mathcal{L}(G^{\mu\nu}(F)\partial_\mu T \partial_\nu T, T)\,. \qquad (2.9)$$

The explicit examples of the actions considered above are all of that type. Then varying (2.9) with respect to A_μ one gets (assuming (2.8))

$$\partial_\mu \left[(I + F)^{-1}_{[\mu\nu]}\, \mathcal{L} + 2(I + F)^{-1}_{1[\mu}(I + F)^{-1}_{\nu]1}(\partial_1 T)^2 \mathcal{L}' \right] = 0\,. \qquad (2.10)$$

Here $\mathcal{L} = \mathcal{L}(K, T)$, $K \equiv G^{\mu\nu}\partial_\mu T \partial_\nu T$ and the derivative $\mathcal{L}'$ is with respect to the first argument K. Since $\mathcal{L}$ depends only on x_1 this equation reduces to

$$(I+F)^{-1}_{[\nu 1]}\,\partial_1[\mathcal{L} - 2G^{11}(\partial_1 T)^2\mathcal{L}'] = 0\,. \tag{2.11}$$

This equation is satisfied if

$$(I+F)^{-1}_{[\nu 1]} = \left(\frac{F}{1-F^2}\right)_{1\nu} = 0\,, \tag{2.12}$$

i.e. if $F_{1\nu} = 0$, *or* if

$$\mathcal{L} - 2G^{11}(\partial_1 T)^2\mathcal{L}' = \text{const}\,. \tag{2.13}$$

The latter condition does not hold for a generic lagrangian, so we will always require that $F_{1\nu} = 0$. Then the effective metric $G^{\mu\nu}$ simplifies, so that

$$G^{11} = 1, \qquad G^{1i} = 0\,. \tag{2.14}$$

Note also that in the axial gauge, $A_1(x_1, x_i) = 0$, the condition $F_{1\nu} = 0$ implies that $A_i(x_i)$ does not depend on x_1, as one would expect for a massless vector field on a D$(d-2)$-brane.

It follows from the translational invariance of the action (2.3) that fluctuations around the soliton solution contain a zero mode $\Phi(x_i)$, which satisfies the equation of motion for a massless scalar in $d-1$ dimensions. As usual (see, e.g., [34]), it is natural to identify this zero mode with a collective coordinate describing fluctuations of the D$(d-2)$-brane in the transverse x_1 direction. For small fluctuations and in the semiclassical approximation, the effective action for the zero mode is obtained by substituting $T \to T(x_1) - \partial_1 T(x_1)\Phi(x_i)$ into (2.3), and integrating over x_1. To describe arbitrary fluctuations in the semiclassical approximation one may then consider the standard ansatz $T \to T(x_1 - \Phi(x_i))$.

However, in general, one should take into account the global SO$(d)_G$ invariance of the action (2.3) implied by the open string sigma model. The subscript G means that the general linear transformations of the d coordinates $x \to \Lambda x$ should preserve the constant metric $G^{\mu\nu}$, i.e.

$$\Lambda G \Lambda^T = G\,.$$

Assuming that Λ has the form

$$\Lambda_{\mu\nu} = \frac{1}{\beta}\tilde{\Lambda}_{\mu\nu}\,, \qquad \tilde{\Lambda}_{11} = 1\,, \qquad \tilde{\Lambda}_{1i} = -V_i\,, \tag{2.15}$$

we get

$$\beta = \sqrt{1 + G^{ij}V_iV_j}\,. \tag{2.16}$$

Consider now the following ansatz for the tachyon in terms of the zero mode $\Phi(x_i)$

$$T = T(y_1)\,, \qquad y_1 \equiv \frac{x_1 - \Phi(x_i)}{\beta(\partial\Phi, F)}\,,$$
$$\beta(\partial\Phi, F) = \sqrt{1 + G^{ij}(F)\partial_i\Phi\partial_j\Phi}\,. \tag{2.17}$$

Note that if $\Phi(x_i)$ is of the form $\Phi(x_i) = V_i x^i$ (i.e. brane has "constant velocity"), then $y_1 = (\Lambda x)_1$ and Λ belongs to the invariance group $\mathrm{SO}(d)_G$.

Taking into account that the full tachyon field in (2.17) depends only on one combination y_1 of coordinates, we can rewrite (2.3) as follows (ignoring higher-derivative $\partial^n \Phi$, $n > 1$ terms)

$$S = T_{d-2} \int d^{d-1}x\, \beta(\partial\Phi, F) \sqrt{\det(\delta_{\mu\nu} + F_{\mu\nu})}\,, \tag{2.18}$$

where the integral over y_1

$$T_{d-2} \equiv \int dy_1 \mathcal{L}\left(T(y_1), \partial_1 T(y_1), \partial_1\partial_1 T(y_1), \ldots\right) \tag{2.19}$$

does not depend on the field strength[9] and thus defines the D$(d-2)$-brane tension.

Taking into account that (for $F_{i1} = 0$)

$$\beta(\partial\Phi, F)\sqrt{\det(\delta_{\mu\nu} + F_{\mu\nu})} = \sqrt{\det(\delta_{ij} + F_{ij} + \partial_i\Phi\partial_j\Phi)}\,, \tag{2.20}$$

we arrive at the standard DBI action for a D$(d-2)$-brane

$$S = T_{d-2} \int d^{d-1}x \sqrt{\det(\delta_{ij} + F_{ij} + \partial_i\Phi\partial_j\Phi)}\ . \tag{2.21}$$

To get the DBI action it was essential to introduce the collective coordinate dependence as in (2.17). Indeed, the above derivation is a generalization of the argument used to show that a point-like soliton of a relativistically invariant (e.g. $2-d$) scalar field theory is described by the standard particle action $M \int dt \sqrt{1-v^2}$: one starts with the static solution and applies a boost $x \to \frac{x-vt}{\sqrt{1-v^2}}$. The ansatz (2.17) of course reduces to $T(x_1) - \partial_1 T(x_1)\Phi(x_i)$ for small fluctuations, but the real reason for the specific non-linear structure of (2.17) is to ensure the decoupling of massive solitonic modes from the massless mode, so that there is no term linear in the massive modes in the action (in the approximation where we neglect second and higher derivatives of Φ). Thus one can consistently put all massive modes to zero, and consider only the Φ-dependent terms.[10] If one would use the standard form of the collective coordinate dependence $T = T(x_1 - \Phi(x_i))$ then to recover the DBI action one would have to take into account the massive mode contributions to the low-energy effective action and at the end to make a proper non-linear redefinition of the collective coordinate Φ.

[9] As follows from the sigma model considerations, $G^{\mu\nu}$ can be contracted only with the tachyon derivative terms. Thus only G^{11} could appear in (2.18), but it is equal to 1 according to (2.14).

[10] This is somewhat similar to what happens in consistent Kaluza-Klein reductions of compactified gravity models.

3. World-sheet picture

Here we shall relate the discussion of the DBI action from the effective field theory point of view in the previous section to the world-sheet sigma model considerations, with the aim to identify the boundary couplings corresponding to the tachyonic soliton collective coordinate.

3.1 Standard boundary couplings

The starting point in the usual perturbative description of D-branes in flat space is an exact free $2-d$ conformal field theory in the bulk with $p+1$ Neumann and $d-p-1$ Dirichlet boundary conditions. The marginal boundary interactions are then represented by the boundary sigma model action [2, 1][11]

$$I_{\partial\Sigma} = \frac{1}{4\pi}\int_{\partial\Sigma} d\tau \Big[\sum_{i=1}^{p+1} A_i(x^1,\ldots,x^{p+1})\dot{x}^i + \sum_{a=p+2}^{d} \Phi_a(x^1,\ldots,x^{p+1})\partial_n x^a\Big]\,, \tag{3.1}$$

where the couplings A_i and Φ_a are the gauge field and the scalars on the world-volume of the Dp-brane, respectively (∂_n is the normal derivative to the boundary $\partial\Sigma$). The open string sigma model partition function on the disc is then the DBI action for A_i and Φ_a (ignoring terms higher than first derivative in the fields).

On the other hand, to describe tachyon condensation from a space-filling brane to a lower-dimensional brane one should start with the following boundary theory

$$I_{\partial\Sigma} = \frac{1}{4\pi}\int_{\partial\Sigma} d\tau [T(x) + A_\mu(x)\dot{x}^\mu]\,. \tag{3.2}$$

In the presence of this interaction the standard Neumann boundary conditions for the open string coordinate fields x^μ are modified to

$$\partial\Sigma: \qquad \partial_n x_\mu + F_{\mu\nu}\dot{x}^\nu + \partial_\mu T = 0\,. \tag{3.3}$$

A particularly simple case for which the open bosonic world-sheet theory remains solvable is obtained by taking A_μ to have a constant field strength and switching on the tachyon with a quadratic profile in one direction, i.e. $T(x) = a + \frac{1}{2}u\, x_1^2$. In this case the boundary conditions are

$$\partial\Sigma: \qquad \partial_n x_1 + F_{1i}\dot{x}_i + u\, x_1 = 0\,, \quad \partial_n x_i + F_{i1}\dot{x}_1 + F_{ij}\dot{x}_j = 0\,. \tag{3.4}$$

For $u=0$ these are the usual Neumann conditions modified by the presence of the constant strength $F_{\mu\nu}$ (and thus the corresponding partition function is the BI action for $F_{\mu\nu}$ with $\mu = 1,\ldots,d$). On the other hand, when the tachyon condenses into the

[11] For the sake of clarity here we shall consider only the bosonic string theory but will ignore tachyonic coupling on the resulting brane.

vacuum $a, u \to \infty$, the field x_1 becomes constrained to vanish at the boundary, i.e. is subject to the Dirichlet boundary condition ($x_1|_{\partial\Sigma} = 0$). This leads to a complete decoupling of F_{1i} components in (3.2), (3.4). Thus, in the IR fixed point one gets the BI action for F_{ij} components only.

Following this approach, it appears that we are missing the dependence on the massless scalar fields describing the transverse motion of the brane. To account for them, one should understand how the normal derivative coupling of the type introduced from the beginning in the standard D-brane description (3.1) is effectively induced in the process of tachyon condensation. This is what we are going to explain below.

3.2 Generalized boundary sigma model and collective coordinate coupling

From the sigma model viewpoint, the role of the tachyonic field is to control a number of "free" (Neumann) space-time dimensions, and in this respect the condensation may be compared with dimensional reduction. However, in the process of reduction the BI action in $p+1$ dimensions does lead to the DBI action in p dimensions with $A_1 \to \Phi$. The procedure of dimensional reduction is of course formally related to T-duality transformation along one world-volume dimension x_1 (implying that A_1 becomes a scalar field describing the transverse fluctuations).

In the world-sheet description, the T-duality transformation simply exchanges the normal and tangential derivatives in the boundary vertex operators [1]. This suggests that in order to make contact with the D-brane description based on (3.1) one may try to generalize the boundary action (3.2) by adding from the beginning an additional normal derivative coupling (with a new coefficient function V_μ)[12]

$$I_{\partial\Sigma} = \frac{1}{4\pi}\int_{\partial\Sigma} d\tau \, [T(x) + A_\mu(x)\dot{x}^\mu + V_\mu(x)\partial_n x^\mu] \, . \tag{3.5}$$

Adding V_μ term may seem irrelevant — in the standard perturbative vacuum where all x^μ are subject to the Neumann boundary conditions this term decouples: computing correlation functions or the disc partition function in perturbation theory in powers of couplings one finds that they do not depend on V_μ.[13]

More precisely, one should determine the boundary conditions dynamically, by minimizing the total string action $I = I_\Sigma + I_{\partial\Sigma}$. Then (depending on a calculation procedure) the normal-derivative term may contribute to the partition function, but this still does not imply the appearance of a new physical degree of freedom associated

[12] Such sigma model and its renormalization was discussed in detail in [37] and references there.

[13] There exists a (point-splitting) regularization preserving the Neumann boundary conditions, and, therefore, any correlation function involving a normal-derivative term vanishes. Note also that lifted to the bulk of the world sheet, this coupling becomes $\int_\Sigma \partial^a(V_\mu(x)\partial_a x^\mu)$, and thus (modulo a term proportional to the x^μ equation of motion) redefines the target space metric by a diffeomorphism-type term.

with V_μ. Indeed, one may interpret such normal derivative couplings as additional pure gauge modes of open string theory, which can be removed by a gauge choice.

Indeed, the general form of the boundary conditions should be

$$\partial\Sigma: \qquad \partial_n x_\mu + N_{\mu\nu}\dot{x}^\nu + N_\mu = 0\,, \tag{3.6}$$

where $N_{\mu\nu}$ and N_μ should be determined from the requirement that the variation of the total string action $\frac{1}{8\pi}\int_\Sigma(\partial x)^2 + I_{\partial\Sigma}$ vanishes. One finds then the following equations[14]

$$\begin{aligned} N_\mu &= \partial_\mu \tilde{T}\,, \qquad & \tilde{T} &\equiv T - N_\nu V^\nu\,, \\ N_{\mu\nu} &= \tilde{F}_{\mu\nu}\,, \qquad & \tilde{F}_{\mu\nu} &\equiv F_{\mu\nu} - \partial_\mu(N_{\rho\nu}V^\rho) + \partial_\nu(N_{\rho\mu}V^\rho)\,. \end{aligned} \tag{3.7}$$

It is then easy to see (cf. (3.6),(3.3)) that one can replace (3.5) by the usual boundary term (3.2) without V_μ but with transformed tachyon $T \to \tilde{T}$ and vector $A_\mu \to \tilde{A}_\mu$ fields:

$$T = (1 + V^\mu\partial_\mu)\tilde{T}\,, \qquad A_\mu = \tilde{A}_\mu - \tilde{F}_{\mu\nu}V^\nu\,. \tag{3.8}$$

Thus adding the normal-derivative term amounts simply to a redefinition of the original open string sigma model couplings in (3.2).

The central point, however, is that in the presence of a nontrivial tachyon condensate the normal derivative coupling becomes relevant — it provides an adequate description of dynamics of the soliton translational zero modes, allowing one to recover the usual D-brane description in the IR.

Assuming that there is a tachyonic condensate breaking translational invariance in x_1-direction, and that V_1 depends only on x_i, one sees that V_1 can be identified with the translational collective coordinate Φ. Indeed, for small V_1

$$\tilde{T} \approx T - V_1\partial_1 T \approx T(x_1 - V_1)\,, \qquad \tilde{A}_i \approx A_i - V_1\partial_1 A_i \approx A_i(x_1 - V_1)\,, \tag{3.9}$$

which is the standard collective coordinate dependence. Another argument for this identification can be given by analyzing the boundary conditions (3.6) in the IR fixed point $a, u \to \infty$ for the quadratic tachyon coupling $T(x_1) = a + \frac{1}{2}ux_1^2$. At this point the boundary condition (3.6) reduces to the following Dirichlet condition for x_1

$$\partial\Sigma: \qquad x_1 = \Phi(x_i)\,, \quad \Phi \equiv V_1(x_i)\,, \tag{3.10}$$

and the modified Neumann condition for x_i (cf. (3.4))

$$\partial\Sigma: \qquad \partial_n x_i + \partial_i\Phi\partial_n x_1 + F_{ij}\dot{x}_j = 0\,. \tag{3.11}$$

The boundary conditions (3.10) and (3.11) define the mixed Dirichlet-Neumann sigma-model. The vanishing of the one-loop β-functions for this model is known [2] to be equivalent to the equations of motion for space-time fields A_i and Φ following from DBI.

[14] These boundary conditions differ from the ones used in [37] and references there.

One may ask then why in order to derive the DBI action in section 2 we needed to use the non-linear ansatz for the tachyon (2.17) instead of the transformed tachyon and gauge fields in (3.9). The redefined tachyon coupling $\tilde{T}$ with the $V_1 = \Phi$ -dependence as in (2.17) can be obtained by starting with the boundary action (3.5) with normal-derivative couplings of an appropriate non-linear form $f(V, \partial_n x, \dot{x})$. This would modify the boundary conditions (3.6) and the V_μ-dependence of the transformed tachyon and vector fields. From the sigma model point of view the difference between the linearized collective coordinate dependence in (3.9) and the non-linear one in (2.17) should be in contact terms only. Such terms are crucial for implementing the symmetries of string theory in a manifest way (see, e.g., [38]), but they do not contribute to the (scheme-independent) one-loop β-functions. This explains why one can reproduce correct β-functions corresponding to the DBI action in the approach of [2] using the linear expressions (3.9). To derive the DBI action as the open string sigma model partition function (which is sensitive to a choice of a scheme preserving underlying symmetries of the theory) one needs, however, to use the non-linear ansatz (2.17) for the collective coordinate dependence, substituting it into the tachyon expansion (2.7) and integrating over the fluctuations ξ^μ.

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